CHAPTER 3.2

The role of models and analogies in the Bohr atom

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Abstract

The significant role of models and analogies in scientific practice has been widely recognized. Modern scholarship on analogy takes its departure from the work of Mary Hesse, who pointed out the existence of negative analogies between two different physical systems, that is, those respects in which the two systems clearly differ. However, she underappreciated the role of negative analogies in model-building. In our paper we will stress the significance of negative analogies for the development of Bohr's atom. We will argue that it was the negative, rather than the positive, analogy between intra-atomic electrons and the rings of Saturn that motivated Bohr to adopt and develop Rutherford's atomic model. The elaboration of the negative analogy led to the conclusion that the electron could move only in certain discrete orbits and its energy and angular momentum were accordingly restricted. Furthermore, a related analogy between electrons and planets played a significant role in Bohr's subsequent articulation of the model. On the one hand, the positive analogy suggested that electrons (like planets) re-

SCI.DAN.M. I · ONE HUNDRED YEARS OF THE BOHR ATOM: PROCEEDINGS FROM A CONFERENCE
THE ROYAL DANISH ACADEMY OF SCIENCES AND LETTERS · 2015

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volved around the center of mass of the atom (solar system). On the other hand, the extremely high speed of electrons (unlike that of planets) suggested that relativity be brought into the picture.

Key words: Niels Bohr; Mary Hesse, Ernest Rutherford, analogy; models; nuclear atom.

1. Models and analogies

1.1. On models

The term "model" is used in a wide variety of ways in philosophy of science, and thus it is difficult to give a comprehensive and precise definition of the term. In the philosophical literature several kinds of scientific models have been identified: iconic or scale models, analogical models, mathematical (or abstract) models. All of these kinds of models are different means of representing physical systems, usually in a simplified and idealized manner. Thus, in what follows we will treat models as (idealized and simplified) representations of physical systems. These representations are often constructed on the basis of analogies between a "target" (the represented system) and a source (an already understood system). These analogies facilitate the (mathematical and verbal) description of the target.[1]

Mary Hesse suggested that models should not be understood as "literal descriptions of nature, but as standing in a relation of analogy to nature."[2] She studied models in connection with analogy and expanded the concept of model to go beyond the purely mechanical representations that were prevalent in the 19th century, arguing that mathematical formalisms could be considered as models too, since they functioned in essentially the same way. There were many hybrid models, "like the Bohr model of the atom in which electrons are conceived to jump discontinuously from one orbit to another, a

1. Psillos (2007), p. 154.
2. Hesse (1953), p. 201.

feat which no mechanical particle can be imagined to perform."[3] This was an important step toward the analysis of theoretical models.[4]

For Achinstein a theoretical model of a target physical system X was taken to be a set of theoretical assumptions (normally of a complex mathematical form) which provided a starting point for the investigation of the system X. Usually the model was constructed under the guidance of the observed similarities between the system X and a known physical system Y (the analogue). The Bohr model of the atom, according to Achinstein, is such a theoretical model, that is, a set of theoretical assumptions that "attributes an underlying mechanism to the hydrogen atom which explains radiation of discrete wavelengths observed when hydrogen is excited."[5] Achinstein noted that there are cases in which the terms "theory" and "model" are used by scientists interchangeably (Bohr's model/theory of the atom): "it is sometimes though not always true that what is called a model is also called a theory, as in the case of the Bohr model of the atom."[6]

Ernan McMullin takes models to be physical structures, different from theories, and suggests that "it is the model, of course, that gives rise to the theory; there is no way for one to hit upon the theory somehow first."[7] McMullin explicitly refers to the Bohr model of the atom as "a very simple physical structure" and he distinguishes it from Bohr's theory: "the Bohr theory is the set of statements describing how such a model would behave in various conditions."[8] In addition, McMullin recognized "that the original model, though

3. Hesse (1953), p. 200.

4. See Achinstein (1965). Giora Hon and Bernard Goldstein (this volume) argue that Bohr thought of his account of the structure of the atom as a theory and not as a model, which he conceived as a mechanical representation. They point out though that after 1914 "the concept of model was extended as a result of including Bohr's theory in the category of model." Throughout this paper, we use the expression "Bohr's model" in this latter more inclusive sense.

5. Achinstein (1968), p. 213.

6. Achinstein (1968), p. 213.

7. McMullin (1968), p. 392.

8. McMullin (1968), p. 392.

suggested by analogies with the planetary system, was not in fact a planetary system";[9] so it would be incorrect to suppose that the planetary model of Kepler and the atomic model of Bohr "are simply different realizations" of the same formal calculus.[10] A close look at "the subsequent history of the model" would reveal the role of differences between the atom and a planetary system: "the subsequent history of the model was governed far more by the differences between the two models than their similarities."[11] As we will argue below, McMullin's insight is borne out by the history of the Bohr atom.

1.2. On analogy

Analogies are widely used in science: in the development and evaluation of scientific theories, in experimental design, and for purposes of instruction and illustration. To understand and explain a novel situation we often resort to what is already familiar. Modern theories of analogy are considerably indebted to the work of Mary Hesse.[12] Analogies (in physics) are relations "either between two hypotheses, or between a hypothesis and certain experimental results, in which certain aspects of both relata can be described by the same mathematical formalism."[13] In certain cases there is an analogy between aspects of a model (e.g., its mathematical structure) and the phenomena explained by means of it: "the most obvious property of a satisfactory model is that it exhibits an analogy with the phenomena to be explained, that is, that there is some identity of structure between the model and the phenomena."[14]

Hesse drew a distinction between three kinds of analogy: positive, negative and neutral. The positive analogy between two different physical systems consists of the properties they have in common. The negative analogy consists of those respects in which the

9. McMullin (1968), p. 395.
10. McMullin (1968), p. 395.
11. McMullin (1968), p. 395.
12. Hesse (1966).
13. Hesse (1953), p. 202.
14. Hesse (1965), p. 102.

two systems clearly differ. Finally, the neutral analogy consists of those properties "of the model about which we do not yet know whether they are positive or negative analogies; these are the interesting properties, because ... they allow us to make new predictions."[15] The neutral analogy, then, plays the most important heuristic role in the development of a theory.

Achinstein has also contributed to understanding the role of analogies in science.[16] Analogies function "by indicating similarities between ... [novel] concepts and others that may be familiar or more readily grasped."[17] Furthermore, they may suggest new principles and facilitate the extension of a theory. For example, similarities between electrostatic and gravitational phenomena indicate that principles governing the system we understand better can be transferred to the other. For instance, it is reasonable to propose an inverse square law for electrostatic attraction on the basis of its analogy with gravitation. Achinstein also observes that the term "analogy" is used in two slightly different senses. First, as indicating "certain types of similarities between two items"; in that sense "analogies are said to exist and to be discovered". Second, analogy refers to "something like a comparison, something one draws, makes, constructs or formulates." But in comparison one looks for similarities and differences too, whereas in analogies "one is looking only for similarities."[18] Thus, for both Hesse and Achinstein, the heuristic function of analogy is exhausted by its positive and neutral aspects. The role of negative analogies, on the other hand, is overlooked.

1.3. How models are related to analogies

Models and analogies are closely connected: models are often based on analogies, and analogies play a crucial role in modeling practices. It remains the case, however, that analogies should not be conflated with models. As Achinstein points out, there is a difference

15. Hesse (1966), p. 8.

16. Achinstein (1964, 1968).

17. Achinstein (1968), pp. 208-209.

18. Achinstein (1968), p. 208.

between a model and an analogy. An analogy is a relationship between two physical systems or entities while a model is usually conceived and constructed on the basis of an analogy. Although many models are formed on the basis of an analogy, most of them outgrow the analogy from which they originated. Bohr's model is characteristic.

We will argue that the negative analogy between the atom and a Saturnian system motivated the creation of Bohr's model of the atom. Other analogies (e.g., the correspondence principle and the analogy between electrons and Planck's oscillators) also played an important role in this respect.[19] Further (positive and negative) analogies between the atom and a planetary system provided important resources for the articulation of the model.

2. Bohr and analogy

The development of models and analogies was a crucial aspect of Bohr's theorizing. He came under the spell of Harald Høffding,[20] his philosophy teacher and a friend of his family, who suggested that analogies play an essential role in science:

> our thinking consists in a comparison of different domains of experience, so that the one can make the other clearer for us. All our knowledge, the spontaneous as well as the scientific, is therefore full of analogies. When thinking proceeds to a new task, it does not take up quite new means and ways, but it tries so far as possible to make use of those which it has already applied, especially if they are clear and plain.[21]

Bohr attended a philosophy course, given by Høffding during his first year at the University of Copenhagen (1904), and he continued to have a very warm relationship with his teacher in the following years. According to Faye, "the strong emphasis Høffding laid on the

19. Darrigol (1992).
20. Cf. Faye (1979, 1988, 1991) and Wise (1990).
21. Høffding (1905), p. 203.

concept of analogy as one of the fundamental methodological principles in science must have influenced Bohr deeply."[22] Furthermore, the development of Bohr's theory of the hydrogen atom was "on the basis of the methodological precept he had learned from Høffding."[23]

In a letter to Rutherford on 21 March 1913, Bohr spoke of his assumptions as suggesting "a possible, very simple way of accounting for a number of facts, and further the most beautiful analogi [sic] between the old electrodynamics and the considerations used in my paper."[24]

3. From Thomson's "plum-pudding" atom to Rutherford's nuclear atom

After he completed his dissertation, Bohr spent some time in Britain and was exposed to the British tradition of model making, which had been "based on the methods of mid-Victorian Cambridge physics."[25] One of the products of that tradition was J. J. Thomson. In 1904 he had proposed a representation of the atom as a positively charged, homogeneous sphere in which electrons revolve in coplanar orbits. Thomson's model promised to account for the periodicities in the chemical properties of the elements and explained adequately the small-angle scattering of β-particles (fast-moving electrons) by matter, as the result of multiple encounters between β-particles and atomic electrons. It was less successful, however, in accounting for the large-angle scattering of α-particles.

In 1909 Hans Geiger and Ernest Marsden reported the results of their experiments on α-scattering. After bombarding a target (a thin sheet of gold) with α-particles, they noticed that a few of those particles were deflected by more than 90°. Rutherford greeted those results with incredulity: "It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit

22. Faye (1991), p. 109.
23. Faye (1991), p. 109.
24. Bohr (1981), p. 584.
25. Heilbron (1981), p. 230; cf. Kragh (2012), p.39.

you."[26] The results, in Rutherford's words "indicate that some of the α particles must suffer a deflexion [sic] of more than a right angle at a single encounter [with an atom]."[27] But for this to occur, "the atom must be a seat of an intense electric field."[28] The electrical forces had to be concentrated in a region of 10^{-13} cm, whereas the size of an atom was of the order of 10^{-8} cm. Rutherford was able to calculate the probability of single scattering at a given angle and found his predictions confirmed experimentally by Geiger and Marsden.

Thus, the results of α-scattering suggested a new representation of the atom, which was "supposed to consist of a central charge surrounded by a uniform distribution of the opposite sign through a sphere of radius R."[29] (Figure 1) For Rutherford, this was a discovery of "what the atom looks like."[30] This image seems to have similarities with the solar system. However, according to his biographer David Wilson, in contrast to what is commonly believed, "this was not the way Rutherford saw the structure." [31] Rutherford started using the word 'nucleus' only about August 1912, and at about the same date seems to have decided that the central charge is positive.[32]

Rutherford's model of the atom was unstable, both mechanically and from an electromagnetic point of view. Rutherford thought that "[t]he question of the stability of the atom proposed need not be considered at this stage, for this will obviously depend on the minute structure of the atom, and on the motion of the constituent charged parts."[33] The scientific community, for the most part, ignored Rutherford's model, which had also nothing to say about two of the most salient contemporary problems: spectra and the periodic table.

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26. Rutherford (1938), p. 68. Heilbron argues that Rutherford's retrospective report of his reaction is exaggerated. See Heilbron (1968), p. 265.
27. Rutherford (1911), p. 669.
28. Rutherford (1911), p. 669.
29. Rutherford (1911), p. 677.
30. Quoted in Wilson (1983), p. 289.
31. Wilson (1983), p. 299.
32. Wilson (1983), p. 305.
33. Rutherford (1911), p. 671.

Figure 1. Rutherford's sketch of the atom in an undated (1910 or 1911) manuscript. Source: H.R. Robinson, "Rutherford: Life and work to the year 1919, with personal reminiscences of the Manchester period," *The Proceedings of the Physical Society* 55 (3, 1943), 161-182, on p. 171.

4. Bohr's quantum twist of the nuclear atom

It was Niels Bohr who further developed Rutherford's model of the atom and made a decisive step towards a new account of atomic structure.[34] Since 1911 and his doctoral work on the electron theory of metals, Bohr had been convinced of the limitations of classical mechanics. Thus, what Bohr found attractive in Rutherford's model, besides its success in accounting for α-scattering, was precisely (and prima facie paradoxically) its mechanical instability. This strengthened his prior conviction that classical mechanics could not give an adequate account of the behavior of individual electrons. This point has been made by Heilbron, who claims that "it is probable that the chief cause of his [Bohr's] conversion was the discovery that the Saturnian atom is mechanically unstable."[35] To put it another way, Bohr's motivation for developing Rutherford's model was the negative analogy between electrons and the rings of Saturn![36]

In early July 1912 Bohr communicated to Rutherford a "memorandum" with his thoughts on the structure of atoms and molecules. He started by outlining Rutherford's model or rather "his own ver-

34. The sketch that follows draws upon chapter 5 of Arabatzis 2006.

35. Heilbron (1977), p. 67. See also Heilbron and Kuhn (1969), p. 241. A ring ("Saturnian") model of the atom had been proposed by the Japanese physicist Hantaro Nagaoka in 1904. By 1909, however, it was forgotten. See Kragh (2012), pp. 23-24.

36. Cf. McMullin (1968), p. 395.

sion of it",[37] as Rutherford had not suggested a ring model of the atom. He stressed the model's difficulties in connection to actual atoms: first, the lack of "stability in the ordinary mechanical sense" for the multi-electron rings. An "equilibrium figuration" demanded "motion of the electrons."[38] This followed from a theorem that had been proved in the early 19th century by Samuel Earnshaw, according to which any system of static particles interacting by inverse square forces is unstable.[39] This constraint was imposed by the model itself (inverse square forces) and the stability of the actual atoms. Second, he noticed that electrons

> can rotate with an infinitely great number of different times of rotation, according to the assumed different radius of the ring; and there seems to be nothing ... to allow from mechanical considerations to discriminate between the different radii and times of vibration.[40]

So the model lacked the resources to determine the size of the atom.

In response to those difficulties, Bohr put forward a hypothesis, "for which there will be given no attempt of a mechanical foundation (as it seems hopeless ...)."[41] He suggested "that there for any stable ring (any ring occurring in the natural atoms) will be a definite ratio between the kinetic energy of an electron in the ring and the time of rotation."[42] Here Bohr drew upon Planck's theory of radiation which quantized the energy of "oscillators" and connected it with their frequency according to the formula $E=hv$. (In the "memorandum", he did not use h, but a constant K (0,6h).) Thus, Bohr attempted to come to terms with the problems of the nuclear model of the atom by imposing a non-mechanical law on the electron's motion.

In July 1913, the first part of Bohr's Trilogy "On the constitution

37. Kragh (2012), p. 51.
38. Bohr (1981), p. 136.
39. Pais (1988), p. 181.
40. Bohr (1981), p. 137.
41. Bohr (1981), p. 137.
42. Bohr (1981), p. 137. It is obvious from a subsequent page of the Rutherford memorandum that Bohr meant the frequency.

of atoms and molecules" was published in the *Philosophical Magazine*. In the introduction he suggested that the above-mentioned difficulties of Rutherford's model could be overcome by introducing "in the laws [of motion of the electrons] ... a quantity foreign to the classical electrodynamics, i.e., Planck's constant."[43] The same maneuver could also fix another difficulty that plagued Rutherford's atom, its radiative instability. According to classical electrodynamics, as a result of their accelerated motion within the atom, electrons should radiate away their energy and collapse on the nucleus. That implication of Rutherford's model, however, was at odds with the behavior of "actual atoms in their permanent state [which] seem to have absolutely fixed dimensions and frequencies."[44] Thus, experimental information about "actual atoms," the target of the analogy, constrained atomic models and led Bohr to modify Rutherford's model, on the basis of Planck's theory of radiation, stating that "the energy radiation from an atomic system does not take place in the continuous way assumed in the ordinary electrodynamics, but that it, on the contrary, takes place in distinctly separated emissions, the amount of energy radiated out from an atomic vibrator of frequency v in a single emission being equal to $\tau h v$ where τ is a whole number, and h is a universal constant."[45]

This novel mechanism of radiation provided the key for "saving" the Balmer formula, a mathematical representation of the structure of the (visible part of the) hydrogen spectrum:[46]

$$\frac{1}{\lambda}=\frac{R}{n_1^2}-\frac{R}{n_2^2}$$

where λ denotes the wavelength of spectral lines, R is a constant (the so-called Rydberg constant), $n_1 = 2$, and $n_2 = 3, 4, \ldots$

Bohr's encounter with the Balmer formula in early February 1913 had been a decisive event in the development of his model of the atom. According to Bohr's recollections, "As soon as I saw

43. Bohr (1913), p. 2.
44. Bohr (1913), p. 4.
45. Bohr (1913), p. 4.
46. Bohr (1922), p. 12.

Balmer's formula ... the whole thing was immediately clear to me."[47] He derived the Balmer formula in three different ways. The most satisfactory of those derivations, which was presented in a lecture to the Danish Physical Society of Copenhagen on 20 December 1913, exhibited an analogy between the structure of the hydrogen spectrum and the structure of its hidden cause, the hydrogen atom. Bohr assumed that the frequency of the emitted radiation during the transition of the electron from a "stationary" orbit with energy E_1 to another stationary orbit with energy E_2 was:[48]

$$\nu = \frac{E_1}{h} - \frac{E_2}{h}$$

Since $\nu = c/\lambda$, where c is the velocity of light, it follows from the Balmer formula that

$$\nu = \frac{Rc}{n_1^2} - \frac{Rc}{n_2^2}$$

From the structural analogy between these two equations, Bohr could derive an expression for the energy levels of the hydrogen atom: "the energy of the system in the nth state, apart from an additive constant, is given by $-\frac{Rhc}{n^2}$."[49]

Another significant aspect of Bohr's model, in the present context, was the idealizations it contained. To facilitate his calculations, Bohr assumed "that the mass of the electron is negligibly small in comparison with that of the nucleus, and further, that the velocity of the electron is small compared with that of light."[50] The relaxation of these assumptions later led to two remarkable successes.

First, in October 1913 Bohr pointed out that, strictly speaking, the electrons and the nucleus revolve around the center of mass of the atom. It follows that in the formula for the Rydberg constant one should replace the mass of the electron by a "reduced mass", equal to $m/(1+m/m_z)$, where m_z is the mass of the nucleus. This cor-

47. Quoted in Heilbron (1985), p. 34.
48. Bohr 1922, p. 11.
49. Bohr (1922), p. 12.
50. Bohr (1913), p. 3.

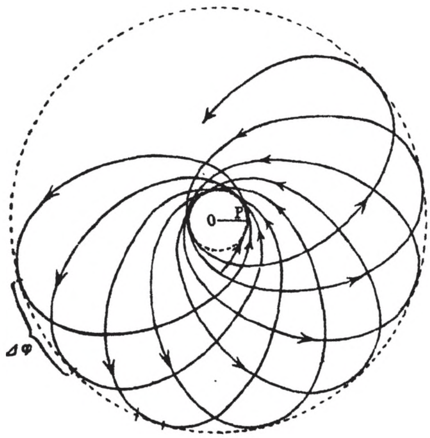

Figure 2. The orbit of the electron in the hydrogen atom according to Sommerfeld. Source: Sommerfeld (1923), p. 467.

rection was strikingly confirmed by spectroscopic measurements.[51] Second, in 1914 it was discovered that Balmer's formula was not entirely accurate.[52] By taking into account, the high speed of electrons within the hydrogen atom and by "replacing the expressions for the energy and the momentum of the electron by those deduced on the theory of relativity,"[53] Bohr showed that the deviations from Balmer's formula could be accommodated. Furthermore, the velocity dependence of the electron's mass made possible an explanation of the so-called fine structure, the doublet structure of most hydrogen spectral lines.[54]

In 1915-1916 Sommerfeld provided a relativistic extension of Bohr's model of the atom, elaborating the negative analogy between electrons and planets.[55] He showed that the variation of the electron's mass as a result of its varying velocity had two effects. First, the orbit of the electron turned out to be a precessing ellipse (Figure 2).

Second, the energies of orbits with the same major axis but different shapes turned out to be different. Thus, by taking into account the velocity dependence of the electron's mass, one was led to

51. Heilbron (1985), p. 35.
52. Curtis (1914).
53. Bohr (1915), p. 334.
54. Bohr (1915), p. 334.
55. Sommerfeld (1916).

a proliferation of the energy levels of the hydrogen atom. Since the lines in the hydrogen spectrum resulted from "difference[s] of energy in the initial and the final path of the electron,"[56] this maneuver, therefore, promised to provide the key for unlocking the riddle of fine structure.

One could view these successes of Bohr's model as the outcome of the positive and negative analogy between electrons and planets. On the positive side, the analogy between electrons and planets suggested that electrons (like planets) revolved around the center of mass of the atom (solar system). On the negative side, the extremely high speed of electrons (unlike that of planets) required that relativity be brought into the picture. In both cases, the analogy played a heuristic role that enabled the enrichment of the original model and its improved fit with empirical data.

5. Concluding remarks

Analogies are of central importance in model-building. Bohr's model of the atom was motivated by the negative analogy between a Saturnian system and the atom. Furthermore, the positive and negative analogies between planets and electrons played a significant role in the subsequent articulation of the model. We emphasized the importance of negative analogies, because they have been neglected by philosophers of science.

One of Bohr's points of departure was the negative analogy between electrons and the rings of Saturn. The former, in contrast to the constituents of the latter, repelled each other and, thus, could not form a mechanically stable system. To fix that defect, Bohr imposed (mechanical and electromagnetic) stability to the atom by fiat. The pursuit of the negative analogy led to the conclusion that the electron could move only in certain discrete orbits and its energy and angular momentum were accordingly restricted. In those orbits the electron was subject to classical mechanics and Coulomb's law, but did not radiate and, thus, defied the laws of classical electrodynamics. It emitted radiation only when it switched orbits,

56. Sommerfeld (1923), p. 67.

and its frequency was specified by a new quantum-theoretical law. This model of the atom was further modified on the basis of another negative analogy, this time between electrons and planets. Their very different speeds implied that their respective orbits should have different characteristics.

Moreover, empirical constraints from spectroscopy motivated, via a positive analogy between the structure of the phenomena and the structure of the atom, a new mechanism of radiation that departed radically from classical physics. Another positive analogy, between atoms and planetary systems, led Bohr to conclude that electrons revolved around the center of mass of the atom (rather than the nucleus).

In all, the Saturnian and planetary models of the atom, under heuristic guidance from the (negative and positive) analogies on which they were based and under pressure from the empirical constraints provided by their target, mediated the transition from a classical to a quantum theory of atomic structure.

Acknowledgments: We would like to thank Finn Aaserud, Helge Kragh, Giora Hon, and an anonymous referee for their helpful comments on an earlier version of this paper. This research has been co-financed by the European Union (European Social Fund - ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF) - Research Funding Program: THALIS - UOA - Aspects and Prospects of Realism in the Philosophy of Science and Mathematics.

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